THE EMPIRICAL DETERMINATION OF AN ÆSTHETIC FORMULA

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I. Previous Investigations

Several attempts have been made in the past few decades to determine the pleasure derived from various types of æsthetic objects by reference to their numerical or geometrical characteristics. Such attempts as those of Emch (5), Birkhoff (2), and Rashevsky (13) may be instanced here. This desire to find a 'formula' for beauty is of course nothing new; Evans has traced it back to the time of the French Revolution (6, p. 51), and it would even appear as if we should place the beginning of this quest in the time of Pythagoras, who was perhaps the first to connect music and number.

However, it is only recently that this type of study has entered into an experimental phase. These experiments have mostly been connected with Birkhoff's *Æsthetic Measure* (2). Birkhoff maintains that our pleasure in any work of art depends on two variables: the amount of Order ('O') and of Complexity ('C') in the object. These are measured in various ways for different classes of objects, but all classes obey the general formula:

$$M \text{ (the amount of pleasure derived)} = O/C.$$

Birkhoff himself has worked out his formula in detail for polygonal figures, vases, poetry, and music. The theory on which the formula is based has been criticised by Davis (4, p. 233), who finds that "we are forced to conclude that the a priori evidence in support of the æsthetic measure formula is insufficient and sometimes dubious."

Several investigators have tested the predictive value of the formula. The results to date are given in Table 1; it will be seen that the correlations between the actual rankings and those given by the formula are positive throughout; there is, however, a great deal of variation in the size of the coefficients, the largest being .70, and the smallest being .05.

TABLE 1

Investigator	Material	Subjects	Correlation with A. M.
R. C. Davis	10 Polygons	162 Students	.11
R. C. Davis	10 Polygons	55 Art Students	.05
F. W. Swift	45 Polygons	6 Laymen	.53
F. W. Swift	45 Polygons	3 Art Students	.22
F. W. Swift	15 Polygons	6 Students	.16
C. M. Harsh et al.	26 Polygons	30 Students	.34
H. J. Eysenck	32 Polygons	14 Observers	.53
H. J. Eysenck	32 Polygons	14 Observers	.62
H. J. Eysenck	15 Polygons	12 Observers	.48
R. C. Davis	10 Poems	63 English Students	.55
H. W. Miller	7 Lines of Poetry	16 English Students	.11
H. W. Miller	7 Lines of Poetry	5 Students	.08
J. B. Parry	19 Lines of Poetry	14 Observers	.34
H. W. Miller	7 Nonsense Lines	5 Students	.68
A. Schnittkind	15 Vases	5 Students	.23
A. Schnittkind	15 Vases	8 Observers	.16
A. Schnittkind	15 Vases	7 Art Students	.09
E. Fischer	10 Chordal Sequences	15 Observers	.05
E. Fischer	5 Unharmonized Melodies	145 Observers	.50
E. Fischer	5 Unharmonized Melodies	59 Observers	.50
E. Fischer	5 Unharmonized Melodies	19 Observers	.70

The investigations by F. W. Swift, H. W. Miller, A. Schnittkind, and E. Fischer are reported by Beebe-Center and Pratt (1). J. B. Parry's investigation has not as yet been reported elsewhere. My own results were the incidental outcome of the investigation described below. The reference to the article by R. C. Davis has already been given.

The most important of these investigations is undoubtedly that carried out by Harsh and Beebe-Center (12). Not content with simply finding the amount of agreement between the formula and the rankings of their subjects, they went further and attempted to discover the factors which influenced the subjects' choice. Using a modified form of Thurstone Factor-Analysis, they found four main factors, which they identified as: (1) Liking for smoothness of contour; (2) Liking for simple, regular geometrical figures; (3) Liking for symmetry, mainly rotational and diagonal; (4) Liking for odd points.

They suggest developing a formula from these four factors, taking into account the extent to which a given observer values each of the judgmental factors. My own approach has been rather different, and it must be said at once that my results agree only partially with those reported above. An attempt to account for the differences will be made at the end of this article.

II. The Present Research

The material used in my main investigation consisted of sixty-four polygons, photographed in black and white from Birkhoff's tables. The principle according to which the polygons were selected was to avoid those which had too obvious associations, such as the swastika or the Jewish star. Otherwise, an attempt was made to include at least one example of each of the various kinds of polygons.

The polygons were divided into two sets, of thirty-two each. The average scores of the two sets by Birkhoff's formula were .39 and .31.

The same fourteen observers were asked to rank the polygons in each of the two sets in order of preference, using a fixed distribution which closely approximated the normal distribution curve. The observers, seven women and seven men, included artists, students, professional men and women, teachers, stenographers, and psychologists. None of them were familiar with Birkhoff's theories.

The rankings in each of the two groups were correlated, and the resulting tables of correlations submitted to a statistical analysis. An effort was made to show that the principal factors in the two tables were identical, and a determination of these factors was attempted.

In order to construct a formula on the basis of the results of these experiments, contingency correlations were worked out between the average order of the polygons, and certain geometrical characteristics which it appeared likely were partly responsible for their relative popularity. The squares of these contingency correlations were then used as weights in a regression equation of the form:

$$M = b_1x_1 + b_2x_2 + b_3x_3 + \cdots + b_nx_n,$$

where the dependent variable, M, is the average judgment; $x_1, x_2, x_3, \cdots x_n$, the independent variables, are the chosen characteristics of the polygons; and $b_1, b_2, b_3 \cdots b_n$ are the weights.

The formula found in this way was tested by correlating the order of the polygons given by it with the average order of the polygons in each of the two sets, and also by applying it to the results of another test.

III. Results and Discussion

In an endeavour to test the stability of the factors from one matrix of correlations to the other, Burt's symmetry criterion was used. This criterion rests on a theorem of frequent use in quantum-mechanics; namely, that the principal axes of two symmetrical matrices, R_1 and R_2, and therefore their principal factors or components, will coincide if, and only if, the two matrices commute, *i.e.*, if

$$R_1R_2 = R_2R_1 = (R_1R_2)',$$

and therefore if the product is itself symmetrical.

When this test is carried out, it is seen that the product-matrix approaches very closely to symmetry; as closely, indeed, as one could reasonably expect in view of the high standard errors. There is no considerable deviation between the sums of the rows and of the corresponding columns; on the average, the deviations are only 3 per cent of the mean totals of the rows and columns, the highest being no more than 5 per cent. Hence we conclude that the principal factors in the two matrices are identical.

Most of the correlations in the two matrices are positive, only ten in one and fourteen in the other being negative, out of ninty-one. The highest correlations in the two matrices are .69 ± .09 and .74 ± .08; the highest negative correlations in both matrices are $-.59 \pm .12$.[1] The averages of the correlations in the two matrices are .27 and .25.

These two values enable us to calculate, by means of a formula which I have shown elsewhere to be applicable to data of this kind, what the correlations of the average orders of the polygons as given by the fourteen observers in the two tests would be with the 'true order,' as given by the whole population of which our observers are only a sample (7, p. 652). These correlations work out at .92 and .91.

Two factors were extracted from each of the two tables of correlations; the first one in each case being a general factor, having positive saturations throughout, the second one being bipolar in each case, *i.e.*, having positive and negative saturations in roughly equal numbers. The general factor accounts for 32 per cent and 31 per cent of the variance, the bipolar factor accounts for 15 per cent and 11 per cent.

In the factorial analysis of correlations between persons, Davies has shown that the second factor to be extracted is nearly always statistically insignificant (3). Employing the same criterion which she used (she tested the residuals on which the second factor was based by Fisher's test of the difference between the theoretical and the actual correlations, expressed in terms of their inverse hyperbolic tangents; $z = \tan h^{-1} r$), the second factor was found to be statistically significant in both tables.

Leaving aside for the moment the first, general factor, we can easily identify the second, bipolar factor by examining the polygons on which there is a great difference of opinion between those subjects with high positive and those with high negative saturations. It would appear that the opposition indicated by this factor is between simple, regular polygons with few sides on the one hand, and complex, not so regular, many-sided polygons on the other. The square, equilateral triangle, rectangle, and diamond are the outstanding examples of the 'simple' type of polygon; some of the polygons with

[1] Following Fisher's advice (11, p. 46), I have given the standard error with each correlation, rather than the probable error.

rotational symmetry, and complicated designs such as the Greek Cross, are representative of the 'complex' type.

As regards the general factor, it would appear to be related to another factor which I have discussed elsewhere (9). In that article, a general factor of æsthetic appreciation was found to run through eighteen tests, each of which involved the ranking of between ten and thirty pictures of such diverse material as portraits, devil masks, mathematical curves, statues, pencil sketches, and pottery. This factor was called 'T,' and was shown to be correlated with factors derived from correlations between rankings of simple colours and of odours.

A test of this 'T'-factor was given to the subjects of the present experiment, and the correlation found between their scores in the test, and their saturations for the general factor in the two polygon tests is definitely significant; it is .68 ± .19. If the 'T'-test were a perfect test of the 'T'-factor, we might say that the polygon-tests are saturated to that extent with 'T,' in a similar way as an intelligence test is said to be saturated with '*g*.'

As regards the nature of this factor, an indication was given in my article (9) as to its possible explanation; I have worked out this suggestion in some detail along the lines of Gestalt theory, and hope to publish it shortly. At the moment, however, our interest lies mainly in those characteristics of the polygons which are correlated positively or negatively with the general factor. Twelve of the most important of these are given in Table 2, together with their contingency correlations with the average order of the polygons.

Some of the terms used in this table are unambiguous; others may stand in need of elucidation. *Vertical and Horizontal Symmetry* are self-explanatory; it should be noted, however, that in this article both these types of symmetry are held to apply only to compact figures, *i.e.*, figures without re-entrant angles, and to semi-compact figures, *i.e.*, figures

[2] This bipolar factor of 'complexity versus simplicity' does not seem to be restricted to polygons; in analysing the rankings of thirty-two poems by fourteen subjects a similar factor was found, dividing those who preferred a regular rhyming scheme, well-marked rhythms, and simplicity of meaning, from those who preferred more experimental, irregular rhyming schemes and rhythms, and greater complexity of meaning.

TABLE 2

	Basis of Judgment	Coefficient of Contingency
(x_1)	Vertical or Horizontal Symmetry	.71
(x_2)	Rotational Symmetry	.69
(x_3)	Equilibrium	.51
(x_4)	Repetition	.45
(x_5)	Compact Figure	.37
(x_6)	Complexity Six or more	.33
(x_7)	Both Vertical and Horizontal Symmetry	.31
(x_8)	Pointed Top and/or Base	.20
(x_9)	Complexity Three or more	.10
(x_{10})	Complexity Two	−.17
(x_{11})	Re-entrant Angles	−.52
(x_{12})	Angles close to 90 degrees or 180 degrees	−.63

having niches the area of which is less than one-sixth of the total area of the polygon. *Rotational Symmetry*, on the other hand, only applies to polygons which are neither compact nor semi-compact. A polygon has *Equilibrium* if the centre of area lies between two vertical lines erected on the extreme points of support on the horizontal base, but at a distance from either of them exceeding one sixth that of the total horizontal breadth of the polygon. It is also said to have equilibrium if it has a pointed base and vertical symmetry. A polygon has *Repetition* if it can be divided into two parts in such a way that one part is the mirror image of the other, while differing from it in size. *Complexity* is defined as the number of non-parallel sides of the polygon. The other terms used will not present any special difficulties.

As explained previously, we now use the squares of the correlations as weights in a regression equation, in order to obtain an empirical formula for the pleasure derived from each polygon. When a few slight adjustments are made, necessitated largely by the fact that one or two of the bases of judgment occur only very rarely in the polygons, and their influence is consequently not represented quite accurately by the coefficient of contingency, the following formula is obtained

$$M = 20x_1 + 24x_2 + 8x_3 + 7x_4 + 5x_5 + 3x_6 + 3x_7 + 2x_8 + 1x_9 - 2x_{10} - 8x_{11} - 15x_{12}.$$

The accuracy of this formula can be tested by correlating the average orders of the two sets of polygons with the orders given by the formula. The coefficients of correlation are respectively .91 ± .03 and .88 ± .04; that is to say, our formula accounts for approximately 80 per cent of all the causal factors entering into the preferential judgments of the polygons included in the two series.

It will be remembered that in view of the average intercorrelations of the subjects' rankings, we concluded that the correlation of their average order with the 'true order' would be about .90. That is to say, even a perfect formula for polygonal forms could not predict the average arrangement more closely than our own formula does. About 20 per cent of the factors entering into the average judgment are chance factors; the other 80 per cent are accounted for by the formula.

Another experiment was carried out in order to see how well this formula would work if applied to the rankings of a different group of observers. Twelve subjects were asked to rank in order of liking fifteen polygons from Birkhoff's selection, five of which had not been used in the two preceding experiments. The rankings were correlated, and two factors extracted from the resulting table. The first, general factor, accounts for 26 per cent of the variance, the second, bipolar factor, for 20 per cent. Again the bipolar factor marks the opposition between liking for 'simple' and for 'complex' polygons.

The average of all the correlations in the table is only .17; that is to say, the correlation of the average order with the 'true order' would be .85. The average order agrees with the order given by the formula to the extent of .84 ± .08. Again nearly all the causal factors, except the chance factors, are accounted for by the formula.

Two points remain to be discussed. One is the relation of the factors found in our analysis to those found by Harsh and Beebe-Center. The other is the rôle to be assigned to the bipolar factor in a formula such as the one given above.

Clearly the results of our analysis are not identical with those obtained by Harsh and Beebe-Center. It may be surmised that this difference is largely due to the method of analysis adopted; Harsh and Beebe-Center would appear to accept Thurstone's theory of primary structure, while the present writer, in conformity with most of the leading British factor-analysts, cannot see his way to accept this theory (8). The fact that a correlation was found between the first or general factor extracted in our analysis, and the 'T'-factor, would seem to point towards some such analysis as ours as being perhaps nearer the truth; but the whole question is not advanced enough at the present moment to be capable of definite settlement.

As regards the bipolar factor, its presence certainly raises a number of problems in connection with the formula. Factors of this kind are often very important, and at times even overshadow the influence of the general factor (*cf.* **10**, for an investigation into bipolar factors). However, by their very nature the two different poles of these factors more or less cancel out, and therefore do not greatly affect the formula.

This is only true when we deal with average judgments. When we attempt to predict the order of liking of any individual, then it becomes necessary in many cases to take into account the bipolar factor. This can be done by extending the procedure used in finding the formula for the general factor to characteristics which differentiate the two poles of the bipolar factor; these can then be weighted for every subject by the saturation (positive or negative) of that subject for the bipolar factor.

Work is in progress at the moment to extend the formula in this way, and it is hoped that substantial progress can soon be reported. An effort is also being made to discover the factors active in the judgments of vases and poems, and to derive formulæ for these two groups of æsthetic objects in a manner similar to that used in the present investigation.

IV. Summary

In an endeavour to test Birkhoff's formula for polygonal forms, and to derive a new formula from the actual experimental results, some seventy polygons were judged by altogether twenty-six observers. Two factors were found to account for all the correlations within the limits of the stand-

ard error: a general, positive factor, which correlated significantly with the 'T'-factor, and a bipolar factor, which divided the people preferring the simple figures from those preferring the complex figures. (A similar bipolar factor had previously been found in judgments of poetry.)

A formula was developed by correlating various possible bases of judgment with the average order of the polygons, and by using the squares of the correlations as weights in a regression equation. This formula was found to account for all the non-chance factors operating in the judgments of the observers in three different groups of polygons.

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