PRIMARY MENTAL ABILITIES.

(PSYCHOMETRIC MONOGRAPHS No. 1.)

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THIS publication is the opening number of a series which the Psychometric Society proposes to issue. It reports the first large experimental inquiry, carried out by the methods of factor analysis described by Thurstone in *The Vectors of the Mind*[1]. The work was made possible by financial grants from the Social Science Research Committee of the University of Chicago, the American Council of Education, and the Carnegie Corporation of New York. The results are eminently worthy of the assistance so generously accorded. Thurstone's previous theoretical account, lucid and comprehensive as it is, is intelligible only to those who have a knowledge of matrix algebra. Hence his methods have become known to British educationists chiefly from the monograph published by W. P. Alexander[8]. This enquiry has provoked a good deal of criticism, particularly from Professor Spearman's school; and differs, as a matter of fact, from Thurstone's later expositions. Hence it is of the greatest value to have a full and simple illustration of his methods, based on a concrete inquiry, from Professor Thurstone himself.

Fifty-six tests, selected according to a provisional classification of cognitive factors, were applied to 240 volunteers. The correlations between the tests were then estimated by means of the charts for tetrachoric correlation previously published by Thurstone and his colleagues.[2] The huge table of correlations has been factorized by the so-called centroid method; and twelve factors extracted. All except the first are bipolar, i.e., have negative as well as positive saturations. It is then assumed that "primary factors act positively unless they are absent from a performance." Hence the co-ordinate axes, representing the factors, are rotated, two at a time, until the negative saturations are virtually obliterated, and the number of zero saturations maximized. For this purpose thirty diagrams have been plotted, and fresh axes fitted by eye.

It is stated that "the graphical method of rotating in one plane at a time is probably the best single method; but the graphical method is not ideal." Although the principles involved have been briefly explained in *The Vectors of the Mind* and elsewhere, hitherto, as Professor Thurstone points out, "there has not been published any adequate description of the method as applied to an actual problem." This,

therefore, is in some ways the most interesting section of the report. The final upshot is thirteen fresh factors, of which nine can readily be given a psychological meaning. In the main, though not in every detail, the interpretations correspond with the categories which the tests were originally selected to represent.

When the editor of this *Journal* first suggested a review of Professor Thurstone's new report, it appeared that the large collection of data contained in its tables would offer an admirable opportunity for testing recent statements about the mode of factor analysis, statements for the most part reached *a priori* and never yet verified by any concrete comparison. How, for example, do Thurstone's methods and results compare with earlier methods and results put forward by workers in this country?

In his 1935 Memorandum ([3], page 306) Burt has pointed out that there are in theory two general ways of factorizing a table of correlations between tests: (*a*) with the first method—a 'submatrix' or 'group factor' method—we may look for relatively specific factors whose influence is solely positive; (*b*) with the second method—a 'general factor' method—we may look for common factors which will be bipolar and therefore have both positive and negative saturations. Where, as in Thurstone's present research, the selection of tests is to a large extent abruptly discontinuous, the former method is evidently the more appropriate. It was, for example, used by Burt and his co-workers nearly twenty years ago in several studies of educational tests, where the subjects tested fall into obviously discontinuous groups. In these early researches the centroid formula (as it is now called) was employed for the first time; and factors very similar to those now reached by Professor Thurstone were elicited ([4], Tables XVIII–XXIV).

The categories which Thurstone's tests were selected to represent are described as follows: (i) Abstraction (Tests 4–8); (ii) Verbal (9–16, to which should obviously be added 56–60 from the 'unclassified'); (iii) Space (17–25); (iv) Number (30–35); (v) Numerical reasoning (36–39); (vi) Verbal reasoning (40–42); (vii) Spatial reasoning (43–45); (viii) Rote-learning (46–51); (ix) Unclassified, including spelling, grammar and vocabulary (52–60). Thurstone's own grouping thus shows how discontinuous his categories are. To a large extent they coincide with well-established group factors. There can, therefore, be little question that the group-factor method is the natural procedure.

Accordingly, it seemed eminently desirable to test this view by applying the group factor method to Thurstone's table of correlations. The formula used is a modification of the simple summation formula (viz., [5], p. 359, equation iv). After eliminating the general factor the remaining

factors are derived from the smaller submatrices of residuals. The only point of difficulty is to determine in advance the lines of division between the several submatrices or clusters, so as to base the general factor on correlations uninfluenced by the one and same group factor. Where the grouping of tests is itself a subject of investigation, we cannot adopt the categories by which the original selection of tests was made; for this would obviously beg the question at issue. The criterion proposed is the degree of resemblance between the various columns of correlations. To study these resemblances we may either calculate the unadjusted inter-columnar correlation or make graphs of the coefficients and judge the resemblances between the contours (cf. 4, fig. 9). Where the correlations between the correlations are non-linear, the latter seems the more reliable as well as the speedier method.

The saturation coefficients obtained by this method are shown in Table I. The first or general factor is responsible for 31 per cent of the variance. On eliminating its effects, there are six submatrices containing significant positive residuals. The group factors derived from these contribute about 2 to 6 per cent of the total variance only. Thus, the general factor is five times as significant as any other.

Professor Spearman, in a paper read at the recent Reading conference, has maintained that Thurstone's table could be fitted by a two-factor analysis and that this procedure would reveal a single general factor. Thurstone, on the other hand, declares: "We cannot report any general common factor in Spearman's sense in the 56 tests that have been analyzed." This is rather surprising, since, in selecting the tests, "special emphasis was laid on those tests which are used as measures of intelligence." Now his Table III does, as a matter of fact, show a 'general common factor in Spearman's sense', i.e., a column of saturation coefficients, all positive, and larger than those in any other column; and its subsequent disappearance is plainly an inevitable result of his method of rotation: this aims, not only at abolishing negative saturations, but also at maximizing the zeros *in every column*, even where the saturations are large and positive throughout. No general factor could survive such a procedure. An analysis by Burt's procedure appears to reconcile the two conclusions: for, with Spearman, we discover a general factor, accountable for more of the total variance than any other, and with Thurstone we discover a number of group-factors having a clear psychological meaning.

In their general nature the group-factors shown in Table I agree almost entirely with those of Professor Thurstone. They prove, indeed, to be much the same as those noted in the earlier researches of Burt and

TABLE I. FACTOR SATURATIONS BY GROUP-FACTOR METHOD.

Test.	*G*	*V*	*L*	*A*	*S*	*C*	*M*	*R*	*Z*
4	·554	·483	—	—	—	—	—	—	—
5	·662	·525	—	—	—	—	—	—	—
9	·293	·531	—	—	—	—	—	—	—
10	·649	·511	—	—	—	—	—	—	—
11	·669	·492	—	—	—	—	—	—	—
16	·611	·437	—	—	—	—	—	—	—
52	·533	·496	—	—	—	—	—	—	—
56	·497	·404	—	—	—	—	—	—	—
58	·398	·832	—	—	—	—	—	—	—
59	·237	·265	—	—	—	—	—	—	—
60	·741	·465	—	—	—	—	—	—	—
12	·605	—	·351	—	—	—	—	—	—
13	·537	—	·548	—	—	—	—	—	—
15	·437	—	·628	—	—	—	—	—	—
57	·688	—	·351	—	—	—	—	—	—
30	·678	—	—	·448	—	—	—	—	—
31	·302	—	—	·649	—	—	—	—	—
32	·395	—	—	·575	—	—	—	—	—
33	·349	—	—	·743	—	—	—	—	—
34	·461	—	—	·641	—	—	—	—	—
35	·565	—	—	·444	—	—	—	—	—
37	·627	—	—	·313	—	—	—	—	—
38	·483	—	—	·465	—	—	—	—	—
39	·683	—	—	·446	—	—	—	—	—
8	·444	—	—	—	·424	—	—	—	—
17	·389	—	—	—	·589	—	—	—	—
18	·495	—	—	—	·606	—	—	—	—
19	·520	—	—	—	·512	—	—	—	—
20	·340	—	—	—	·750	—	—	—	—
21	·670	—	—	—	·489	—	—	—	—
22	·504	—	—	—	·622	—	—	—	—
23	·510	—	—	—	·497	—	—	—	—
24	·565	—	—	—	·453	—	—	—	—
27	·367	—	—	—	·555	—	—	—	—
28	·575	—	—	—	·382	—	—	—	—
29	·561	—	—	—	·336	—	—	—	—
36	·304	—	—	—	·214	—	—	—	—
45	·696	—	—	—	·325	—	—	—	—
53	·299	—	—	—	·525	—	—	—	—
6	·814	—	—	—	—	·436	—	—	—
7	·684	—	—	—	—	·364	—	—	—
14	·657	—	—	—	—	·427	—	—	—
26	·418	—	—	—	—	·549	—	—	—
51	·309	—	—	—	—	·445	—	—	—
46	·361	—	—	—	—	—	·499	—	—
47	·527	—	—	—	—	—	·569	—	—
48	·420	—	—	—	—	—	·457	—	—
49	·472	—	—	—	—	—	·404	—	—
50	·370	—	—	—	—	—	·495	—	—
40	·688	—	—	—	—	—	—	[·575]	—
42	·653	—	—	—	—	—	—	[·575]	—
54	·409	—	—	—	—	—	—	—	[·520]
55	·707	—	—	—	—	—	—	—	[·520]
25	·584	—	—	—	—	—	—	—	—
41	·824	—	—	—	—	—	—	—	—
43	·868	—	—	—	—	—	—	—	—
44	·772	—	—	—	—	—	—	—	—
Per cent Variance	30·80	5·00	1·65	4·58	6·61	1·74	1·79	[1·16]	[·097]

Factor *V*—Verbal-Literary.
Factor *L*—Verbal-Linguistic.
Factor *A*—Arithmetical.
Factor *S*—Visuo-Spatial.
Factor *C*—Classification.
Factor *M*—Memory.
Factor *R*—Relational.
Factor *Z*—Audio-Rhythmic.
Factor *G*—General Factor of Mental Ability.

his co-workers on London school children: there he found, in addition to the general factor, more or less identifiable with 'intelligence', two verbal, one arithmetical, a manual, and (in tests more purely psychological) a factor for memory and a factor or factors for sensory perception. Here no manual factor is discovered: but that is presumably because in a collection of tests to be given by the group procedure Thurstone was unable to include any tests of manual dexterity or skill; the place of the manual or mechanical factor seems largely taken by the spatial factor. But perhaps the most interesting point of agreement between the present table and the earlier results is the presence of *two* distinguishable verbal factors: this moreover accords, not only with the conclusions drawn in the London work ([4], p. 59), but also, it would seem, with Thurstone's own conclusion. The only important discrepancy between Thurstone's list and ours is that he distinguishes three types of relational or rational factors, whereas we find hardly any significant evidence for one. The reason is clear. If (as Burt has maintained) intelligence is manifested most fully and most clearly in 'activities involving reasoning, i.e., the use of logical relations' ([7], p. 12), then Thurstone's 'logical factors' are mainly a special manifestation of our general factor. Thurstone does not refer to Alexander's work ([8]; cf. 9, pp. 365–71): but it may be noted that Alexander, who used a similar method of rotation, also confirmed the existence of a general, a verbal, an arithmetical, and a practical factor, and endeavoured to demonstrate their importance for educational and vocational practice.[1]

Perhaps, however, the most interesting result of our analysis is this. By the use of a very simple procedure we are able to demonstrate and calculate much the same factors as are demonstrated and calculated by Thurstone. Thurstone's own analysis depends first on making an elaborate formal analysis by the centroid method and then rotating the

[1] Since the foregoing analysis was undertaken, we have learnt that Professor Holzinger has also made an analysis of Thurstone's data on somewhat similar lines[9]. The volume of *Psychometrika*[9] containing this study was not received by our Department until late in the year; hence our investigation was taken in complete independence of Holzinger's. As has elsewhere been pointed out ([5], p. 361), Holzinger's *new* method of bi-factor analysis (not his original method) is in general principle largely identical with Burt's earlier group-factor method. The chief differences are, first, that Holzinger allocates the tests on the basis of what he calls a beta-coefficient, and, secondly, that his method of deducing the general factor saturations would appear to depend on a multiplication formula rather than on a summation formula. Neither in his *Student Manual* nor in his previous Reports of the Spearman-Holzinger Trait Committee does he express his method in terms of an actual formula; but the method as described would appear to imply the use of Burt's equation vi instead of his equation iv ([5], p. 355). In spite of these slight divergences in procedure, our results appear to be closely similar. In each group, however (except those for arithmetic and memory), our own table shows one or two minor additions and one or two omissions as compared with Holzinger's. It may be added that our method, with 9 factors, accounts for more of the total variance than Holzinger's with 10.

axes thus found by a somewhat prolonged and admittedly precarious graphical procedure. The submatrix method reaches the same results directly with one set of simple calculations. Since we have relied on fewer factors, our figures do not fit the observed correlations quite so well as Thurstone's. But of the residuals remaining from our analysis only 2 out of 1,596 are over 0·3. When, as here, the probable errors are high ($\pm 0{\cdot}07$ according to Thurstone), residuals of this size can have no statistical significance, particularly in so huge a table. If a more complete set of saturations were required, giving a slightly closer fit, it could be obtained by carrying the calculation a stage further according to the method described and illustrated in a previous number of this *Journal* ([6], p. 55).

To educationists one of the most interesting chapters in the monograph is the last. This deals with the uses of mental ' profiles ' based on the factor measurements, and suggests the possibility of picking out those individuals who are marked by exceptionally high or exceptionally low performance in some particular factor and therefore might be said to belong to the ' type ' which that factor designates. In particular, it is found that many of the individual profiles show an instructive relation to the vocational interests and wishes of the persons they represent: thus the two youths having profiles with the highest relative scores in the factor of verbal relations (what we have called the verbal literary factor) desire to be teachers; others, who have high scores in the visio-spatial factor, wish to be engineers or geologists.

In conclusion we must express our admiration for the great care and thoroughness which has evidently been expended upon this research. It is, indeed, one of the most valuable educational experiments of its kind hitherto carried out. It provides a mass of figures for those who wish to test alternative methods of analysis; and anyone who wishes to be acquainted with the factorial technique in educational research will find this book a most lucid and instructive introduction.

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